Bound-State β^-–Decay of Free Neutron

Mario Pitschmann

Atominstitut der Österreichischen Universitäten, Technische Universität Wien, Österreich

Talk at 7th Workshop on Ultra Cold and Cold Neutrons, Physics and Sources

June 8 - 14, 2009 / St. Petersburg - Russia

The Bound-State β^-–Decay of Free Neutron

- Symmetries of Weak Interactions accessible by precise measurement of correlation of Neutron spin with spin and momentum of Decay Products
- Elegant method to measure precisely relative spin alignments of the daughter products:
  - Bound-State β^-–Decay: $n \to \mathrm{H} + \nu_e^*$
- Angular Distributions of Probabilities of Bound-State Decay for Experimental Analysis of Contributions of Scalar and Tensor Weak Interactions
- References:
  - W. Schott *et al.*, Eur. Phys. J. A **30**, 603 (2006)
  - T. Faestermann *et al.*, A talk at EXA08 Conference, 15-18 September, SMI Vienna (2008)

Hamiltonian of $V-A$ Weak Interactions

$$\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} V_{ud}\,[\bar{\psi}_p(x)\gamma_\mu(1-g_A\gamma^5)\psi_n(x)]\,[\bar{\psi}_e(x)\gamma^\mu(1-\gamma^5)\psi_{\nu_e}(x)]$$

Parameters of Weak Interactions

- Fermi Constant: $G_F = \frac{g_W^2}{8m_W^2} = 1.1664 \times 10^{-11}\,\mathrm{MeV}^{-2}$
- CKM Matrix Element: $V_{ud} = 0.97419(22)$
- Axial Coupling Constant: $g_A = 1.2750(9)$

From Continuum-...

$$|\mathrm{p}(\vec{k}_p)\mathrm{e}^-(\vec{k}_e)\rangle = a_e^\dagger(\vec{k}_e,\sigma_e)a_p^\dagger(\vec{k}_p,\sigma_p)|0\rangle$$

...to Bound-State β^-–Decay...

$$|\mathrm{H}^{(ns)}(\vec{q})\rangle = \frac{1}{(2\pi)^3}\sqrt{2E_H(\vec{q})}\int\frac{d^3k_e}{\sqrt{2E_e(\vec{k}_e)}}\frac{d^3k_p}{\sqrt{2E_p(\vec{k}_p)}}\delta^{(3)}(\vec{q}-\vec{k}_e-\vec{k}_p)$$
$$\phi_{(ns)}\left(\frac{m_p\vec{k}_e-m_e\vec{k}_p}{m_p+m_e}\right)a_e^\dagger(\vec{k}_e,\sigma_e)a_p^\dagger(\vec{k}_p,\sigma_p)|0\rangle$$

with Schrödinger-Hydrogen Wave function $\phi_{(ns)}$ in momentum representation, since Hyperfine Splitting and States $\ell > 1$ are negligible

Amplitude of the Bound-State Neutron Decay in the Non-Relativistic Approximation for Neutron and Hydrogen

$$M(n \to \mathrm{H}^{(ns)} + \tilde{\nu}_e) =$$

$$G_F V_{ud} \sqrt{2m_n\, 2E_{\mathrm{H}}\, 2E_{\tilde{\nu}_e}} \int \frac{d^3k}{(2\pi)^3}\, \phi^*_{ns}\Big(\vec{k} - \frac{m_e}{m_p + m_e}\,\vec{q}\,\Big)$$

$$\times \Big\{[\varphi^\dagger_p \varphi_n]\cdot[\varphi^\dagger_e \chi_{\tilde{\nu}_e}] - g_A\,[\varphi^\dagger_p \vec{\sigma}\,\varphi_n]\cdot[\varphi^\dagger_e \vec{\sigma}\,\chi_{\tilde{\nu}_e}]\Big\}$$

Since for the Wave function...

$$\int \frac{d^3k}{(2\pi)^3}\, \phi^*_{ns}\Big(\vec{k} - \frac{m_e}{m_p + m_e}\,\vec{q}\,\Big) = \psi^*_{ns}(0) = \sqrt{\frac{\alpha^3 m_e^3}{n^3 \pi}}$$

...the Amplitude becomes

$$M(n \to \mathrm{H}^{(ns)} + \tilde{\nu}_e) = G_F V_{ud} \sqrt{2m_n\, 2E_{\mathrm{H}}\, 2E_{\tilde{\nu}_e}} \sqrt{\frac{\alpha^3 m_e^3}{n^3 \pi}}$$
$$\times \left\{ [\varphi_p^\dagger \varphi_n] \cdot [\varphi_e^\dagger \chi_{\tilde{\nu}_e}] - g_A\, [\varphi_p^\dagger \vec{\sigma}\, \varphi_n] \cdot [\varphi_e^\dagger \vec{\sigma}\, \chi_{\tilde{\nu}_e}] \right\}$$

The probability $P_{(ns)}$ for the population of the (ns) state

$$P_{(ns)} = \frac{1}{\zeta(3) n^3} \quad \textbf{with} \quad \zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3} = 1.202$$

$$P_{(1s)} = 83.19\%$$
$$P_{(2s)} = 10.40\%$$
$$P_{(\geq 3s)} = 6.41\%$$

Bound-State β^-–Decay Rate of Free Neutron

$$\lambda_{\beta_b^-} = \frac{1}{2m_n}\int \frac{1}{2}\sum_{n=1}^{\infty}\sum_{\sigma_n,\sigma_p,\sigma_e}|M(n\to \mathrm{H}^{(ns)}+\tilde{\nu}_e)|^2$$

$$\times\,(2\pi)^4\delta^{(4)}(k_{\tilde{\nu}_e}+q-p)\,\frac{d^3q}{(2\pi)^3 2E_{\mathrm{H}}}\,\frac{d^3k_{\tilde{\nu}_e}}{(2\pi)^3 2E_{\tilde{\nu}_e}}$$

becomes

$$\lambda_{\beta_b^-} = (1+3g_A^2)\,\zeta(3)\,G_F^2|V_{ud}|^2\,\frac{\alpha^3 m_e^3}{\pi^2}\sqrt{(m_p+m_e)^2+Q^2_{\beta_c^-}}\;\frac{Q^2_{\beta_c^-}}{m_n}$$

with Q-value

$$Q_{\beta_c^-} = \frac{m_n^2-(m_p+m_e)^2}{2m_n} = 0.782\,\mathrm{MeV}$$

Ratio of the Continuum and Bound-State β^-–Decay Rates

$$R_{b/c} = \lambda_{\beta^-_b}/\lambda^{(\gamma)}_{\beta^-_c}$$
$$= 2\pi\zeta(3)\,\frac{\alpha^3 m_e^3 Q^2_{\beta^-_c}}{m_n}\,\frac{\sqrt{(m_p+m_e)^2+Q^2_{\beta^-_c}}}{f^{(\gamma)}(Q_{\beta^-_c}, Z=1)}$$
$$= 3.92\times 10^{-6}$$

with the Fermi Integral $f^{(\gamma)}(Q_{\beta^-_c}, Z=1)$ containing radiative corrections
($\rightarrow$ Talk by Andrei Ivanov: Continuum-State β^-–Decay of Free Neutron)

$$\frac{\tau_{\beta^-_b}}{\tau_{\beta^-_c}} = \frac{\lambda_{\beta^-_c}}{\lambda_{\beta^-_b}} \simeq \frac{7.11\,\mathrm{y}}{880\ \mathrm{s}}$$

Hamiltonian of $V - A$ Weak Interactions including Scalar and Tensor Contributions

$$\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} V_{ud} \Big\{ [\bar{\psi}_p(x)\gamma_\mu(1 - g_A\gamma^5)\psi_n(x)]\,[\bar{\psi}_e(x)\gamma^\mu(1-\gamma^5)\psi_{\nu_e}(x)] + g_S\,[\bar{\psi}_p(x)\psi_n(x)]\,[\bar{\psi}_e(x)(1-\gamma^5)\psi_{\nu_e}(x)] + \frac{1}{2} g_T\,[\bar{\psi}_p(x)\sigma_{\mu\nu}\gamma^5\psi_n(x)]\,[\bar{\psi}_e(x)\sigma^{\mu\nu}(1-\gamma^5)\psi_{\nu_e}(x)] \Big\}$$

Amplitude of the Bound-State Neutron Decay in the Non-Relativistic Approximation for Neutron and Hydrogen

$$M^{(ST)}(n \to \mathrm{H}^{(ns)} + \tilde{\nu}_e) =$$

$$G_F V_{ud} \sqrt{2m_n\, 2E_{\mathrm{H}}\, 2E_{\tilde{\nu}_e}} \int \frac{d^3k}{(2\pi)^3}\, \phi^*_{ns}\Big(\vec{k} - \frac{m_e}{m_p + m_e}\,\vec{q}\,\Big)$$

$$\times \Big\{(1 + g_S)\,[\varphi^\dagger_p \varphi_n]\cdot[\varphi^\dagger_e \chi_{\tilde{\nu}_e}] - (g_A + g_T)\,[\varphi^\dagger_p \vec{\sigma}\,\varphi_n]\cdot[\varphi^\dagger_e \vec{\sigma}\,\chi_{\tilde{\nu}_e}]\Big\}$$

Since for the Wavefunction...the Amplitude becomes

$$M^{(ST)}(n \to \mathrm{H}^{(ns)} + \tilde{\nu}_e) = G_F V_{ud} \sqrt{2m_n\, 2E_{\mathrm{H}}\, 2E_{\tilde{\nu}_e}} \sqrt{\frac{\alpha^3 m_e^3}{n^3 \pi}}$$

$$\times \Big\{(1 + g_S)\,[\varphi^\dagger_p \varphi_n]\cdot[\varphi^\dagger_e \chi_{\tilde{\nu}_e}] - (g_A + g_T)\,[\varphi^\dagger_p \vec{\sigma}\,\varphi_n]\cdot[\varphi^\dagger_e \vec{\sigma}\,\chi_{\tilde{\nu}_e}]\Big\}$$

The population probability $P_{(ns)}$ is clearly not affected

Bound-State β^-–Decay Rate of Free Neutron

$$\lambda^{(ST)}_{\beta^-_b} = \Big((1+g_S)^2 + 3\,(g_A+g_T)^2\Big)\,\zeta(3)\,G_F^2|V_{ud}|^2 \times \frac{\alpha^3 m_e^3}{\pi^2}\sqrt{(m_p+m_e)^2+Q^2_{\beta^-_c}}\,\frac{Q^2_{\beta^-_c}}{m_n}$$

Neglecting Quadratic Terms in g_S and g_T

$$\lambda^{(ST)}_{\beta^-_b} = (1+b)\,\lambda_{\beta^-_b}$$

with the Fierz Term defined for the Continuum β^-–Decay (→ Talk by Andrei Ivanov: Continuum-State β^-–Decay of the Free Neutron)

$$b = 2\,\frac{g_S + 3g_A g_T}{1+3g_A^2} = 0.0032(23)$$

Ratio of the Continuum and Bound-State β^-–Decay Rates

$$R^{(ST)}_{b/c} = \lambda^{(ST)}_{\beta^-_b} / \lambda^{(ST/\gamma)}_{\beta^-_c} = \frac{(1+b)\,\lambda_{\beta^-_b}}{(1+b\,\Delta_F)\,\lambda^{(\gamma)}_{\beta^-_c}}$$
$$\simeq \Big(1 + (1-\Delta_F)\,b\Big) R_{b/c}$$
$$= 3.92 \times 10^{-6}$$

with $\Delta_F = \tilde{f}^{(\gamma)}(Q_{\beta^-_c}, Z=1)/f^{(\gamma)}(Q_{\beta^-_c}, Z=1) = 0.6612$
($\rightarrow$ Talk by Andrei Ivanov: Continuum-State β^-–Decay of the Free Neutron)

Ratio of the Continuum and Bound-State β^-–Decay Rates

$$R_{b/c} = 4.20 \times 10^{-6}$$

obtained in

- J. N. Bahcall, Phys. Rev. **124**, 495 (1961)
- P. K. Kabir, Phys. Lett. B **24**, 601 (1967)
- L. L. Nemenov, Sov. J. Nucl. Phys. **31**, 115 (1980)
- X. Song, J. Phys. G: Nucl. Phys. **13**, 1023 (1987)

deviates from our ratio $R_{b/c} = 3.92 \times 10^{-6}$ by 7%

Helicity Amplitudes and Angular Distributions

Antineutrino Spin Wave function for polar angle ϑ between Neutron quantisation axis and Antineutrino momentum

$$\chi_{\tilde{\nu}_e} = \begin{pmatrix} -e^{-i\varphi} \sin\frac{\vartheta}{2} \\ \cos\frac{\vartheta}{2} \end{pmatrix}$$

where φ is the azimuthal angle
(Helicity of Antineutrino is +1, i.e. spin direction is parallel to momentum)

Evaluation of

$$f_{\sigma_n,\sigma_p,\sigma_e,\sigma_{\tilde{\nu}_e}} = (1 + g_S)\,[\varphi_p^{\dagger}\varphi_n]\cdot[\varphi_e^{\dagger}\chi_{\tilde{\nu}_e}] - (g_A + g_T)\,[\varphi_p^{\dagger}\vec{\sigma}\,\varphi_n]\cdot[\varphi_e^{\dagger}\vec{\sigma}\,\chi_{\tilde{\nu}_e}]$$

for all possible spin states is reproduced in Table:

$V - A$, Scalar and Tensor Weak Interactions

σ_n	σ_p	σ_e	$\sigma_{\tilde{\nu}_e}$	$f_{\sigma_n,\sigma_p,\sigma_e,\sigma_{\tilde{\nu}_e}}$
$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$(1 + g_S + g_A + g_T)\cos\frac{\vartheta}{2}$
$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-(1 + g_S - g_A - g_T)\,e^{-i\varphi}\sin\frac{\vartheta}{2}$
$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0
$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-2\,(g_A + g_T)\cos\frac{\vartheta}{2}$
$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$2\,(g_A + g_T)\,e^{-i\varphi}\sin\frac{\vartheta}{2}$
$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0
$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$(1 + g_S - g_A - g_T)\cos\frac{\vartheta}{2}$
$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-(1 + g_S + g_A + g_T)\,e^{-i\varphi}\sin\frac{\vartheta}{2}$

The helicity amplitudes

$$M(n \to H_{FM_F} + \bar{\nu}_e)_{\sigma_n,\sigma_\nu}\ ,$$

read in terms of $f_{\sigma_n,\sigma_p,\sigma_e,\sigma_{\tilde{\nu}_e}}$ for neutron spin up

$$M(n \to H_{00} + \bar{\nu}_e)_{+\frac{1}{2},+\frac{1}{2}} \propto \frac{1 + 3g_A + g_S + 3g_T}{\sqrt{2}}\cos\frac{\vartheta}{2}$$

$$M(n \to H_{11} + \bar{\nu}_e)_{+\frac{1}{2},+\frac{1}{2}} \propto (1 - g_A + g_S - g_T)e^{-i\varphi}\sin\frac{\vartheta}{2}$$

$$M(n \to H_{10} + \bar{\nu}_e)_{+\frac{1}{2},+\frac{1}{2}} \propto \frac{1 - g_A + g_S - g_T}{\sqrt{2}}\cos\frac{\vartheta}{2}$$

$$M(n \to H_{1-1} + \bar{\nu}_e)_{+\frac{1}{2},+\frac{1}{2}} \propto 0$$

...and

$$M(n \to H_{FM_F} + \bar{\nu}_e)_{\sigma_n,\sigma_\nu}$$

in terms of $f_{\sigma_n,\sigma_p,\sigma_e,\sigma_{\tilde{\nu}_e}}$ for neutron spin down

$$M(n \to H_{00} + \bar{\nu}_e)_{-\frac{1}{2},+\frac{1}{2}} \propto \frac{1 + 3g_A + g_S + 3g_T}{\sqrt{2}}\, e^{-i\varphi} \sin\frac{\vartheta}{2}$$

$$M(n \to H_{11} + \bar{\nu}_e)_{-\frac{1}{2},+\frac{1}{2}} \propto 0$$

$$M(n \to H_{10} + \bar{\nu}_e)_{-\frac{1}{2},+\frac{1}{2}} \propto -\frac{1 - g_A + g_S - g_T}{\sqrt{2}}\, e^{-i\varphi} \sin\frac{\vartheta}{2}$$

$$M(n \to H_{1-1} + \bar{\nu}_e)_{-\frac{1}{2},+\frac{1}{2}} \propto (1 - g_A + g_S - g_T) \cos\frac{\vartheta}{2}$$

Angular Distributions of the Probabilities $R_F^{(\pm)}$:

$$4\pi\,\frac{R_F^{(\pm)}}{d\Omega} = \frac{|f_{\pm,\sigma_p,\sigma_\ell,+}|^2}{\displaystyle\sum_{\sigma_n,\sigma_p,\sigma_\ell} |f_{\sigma_n,\sigma_p,\sigma_\ell,+}|^2} \simeq \frac{1}{2}\,\frac{|f_{\pm,\sigma_p,\sigma_\ell,+}|^2}{(1+3g_A^2)(1+b)}$$

Results (→ next frame) agree in the limit $g_S = g_T = 0$ with

- X. Song, J. Phys G: Nucl. Phys. **13** 1023 (1987)

Results for $F = 0$:

$$4\pi\,\frac{R^{(+)}_{F=0}}{d\Omega} = \frac{1}{8}\,\frac{(1+3g_A)^2}{(1+3g_A^2)(1+b)}\left(1+2\,\frac{g_S+3g_T}{1+3g_A}\right)(1+\cos\vartheta)$$

$$4\pi\,\frac{R^{(-)}_{F=0}}{d\Omega} = \frac{1}{8}\,\frac{(1+3g_A)^2}{(1+3g_A^2)(1+b)}\left(1+2\,\frac{g_S+3g_T}{1+3g_A}\right)(1-\cos\vartheta)$$

Results for $F = 1$:

$$4\pi\,\frac{R^{(+)}_{F=1}}{d\Omega} = \frac{1}{8}\,\frac{(1-g_A)^2}{(1+3g_A^2)(1+b)}\left(1+2\,\frac{g_S-g_T}{1-g_A}\right)(3-\cos\vartheta)$$

$$4\pi\,\frac{R^{(-)}_{F=1}}{d\Omega} = \frac{1}{8}\,\frac{(1-g_A)^2}{(1+3g_A^2)(1+b)}\left(1+2\,\frac{g_S-g_T}{1-g_A}\right)(3+\cos\vartheta)$$

$V - A$, Scalar and Tensor Weak Interactions

- **For** $\sigma_n = +\frac{1}{2}, \vartheta = \pi$
- **and** $\sigma_n = -\frac{1}{2}, \vartheta = 0$

only Hydrogen in Hyperfine state with $F = 1$ can be detected
(no "backward" scattering of $F = 0$ states due to angular momentum conservation)

Approximate Numerical Values:

$$R_{F=0} = \sum_{\sigma_n} \int d\Omega \frac{R^{(\sigma_n)}_{F=0}}{d\Omega} = 0.9872\Big(1 + 2\,\frac{g_S + 3g_T}{1 + 3g_A}\Big)\Big|_{\binom{g_S=-0.0251}{g_T=+0.0090}} = 0.99$$

$$R_{F=1} = \sum_{\sigma_n} \int d\Omega \frac{R^{(\sigma_n)}_{F=1}}{d\Omega} = 0.0096\Big(1 + 2\,\frac{g_S - g_T}{1 - g_A}\Big)\Big|_{\binom{g_S=-0.0251}{g_T=+0.0090}} = 0.01$$

Summary: arXiv: 0906.0959 [hep-ph]

- We have calculated the bound-state β^-–decay rate of the neutron relative to the continuum-state β^-–decay rate of the neutron and the angular distributions of the probabilities of the bound-state β^-–decay of the polarised neutron as functions of axial, scalar and tensor coupling constants.
- We have shown that the ratio of the bound-state β^-–decay rate to the continuum-state β^-–decay rate does not depend practically on the values of the axial, scalar and tensor coupling constants and equal to $R_{b/c} = 3.92 \times 10^{-6}$.
- We have shown that the angular distributions of the probabilities of the bound-state β^-–decay of the polarised neutron can be very useful for the measurements of the decays of the neutron into hydrogen in the hyperfine states with F = 1.

The results, expounded in this talk, are obtained in Collaboration with

- Manfried Faber, Atomic Institute of the Austrian Universities, TU Wien, Vienna, Austria
- Andrei Ivanov, Atomic Institute of the Austrian Universities, TU Wien, Vienna, Austria
- Violetta Ivanova, State Polytechnic University of St. Petersburg, St. Petersburg, Russia
- Johann Marton, Stefan Meyer Institute of Austrian Academy of Sciences, Vienna, Austria
- Anatoly Serebrov, Petersburg Nuclear Physics Institute of Russian Academy of Sciences, St. Petersburg, Russia
- Natalia Troitskaya, State Polytechnic University of St. Petersburg, St. Petersburg, Russia,
- Maximilian Wellenzohn, Atomic Institute of the Austrian Universities, TU Wien, Vienna, Austria

Thank You For Attention