Continuum-State β^-–Decay of Free Neutron

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Modern Status of Continuum-State β^-–Decay of Free Neutron

Lifetime of Neutron: $\tau_{\beta^-_c}$

$$\tau^{\rm exp}_{\beta^-_c} = 878.5(8)\,{\rm s} \quad \text{A. P. Serebrov } et\ al.,\ \text{PRC } \mathbf{78},\ 035505\ (2008)$$

$$\tau^{\rm exp}_{\beta^-_c} = 885.7(8)\,{\rm s} \quad \text{C. Amsler } et\ al.,\ \text{PLB } \mathbf{667},\ 1\ (2008)$$

$$g_A = 1.2695(29) \quad \text{Particle Data Group } 2004 - 2008$$

Neutron Spin-Electron Correlation:

$$A = -2\,g_A(g_A - 1)/(1 + 3g^2_A)$$

$$A^{\rm exp} = -0.11933(34) \quad \text{H. Abele, Progr. Part. Nucl. Phys., } \mathbf{60}, 1(2008)$$

$$A^{\rm exp} = -0.11933(34) \longrightarrow g_A = 1.2750(9)$$

$$g_A = 1.2695(29) \longrightarrow A = -0.11727(109)$$

Standard $V - A$ Theory of Weak Interactions

Hamiltonian of $V - A$ Weak Interactions

$$\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} V_{ud}\,[\bar{\psi}_p(x)\gamma_\mu(1 - g_A\gamma^5)\psi_n(x)]\,[\bar{\psi}_e(x)\gamma^\mu(1-\gamma^5)\psi_{\nu_e}(x)]$$

Parameters of Weak Interactions

- Fermi Constant: $G_F = \frac{g_W^2}{8m_W^2} = 1.1664 \times 10^{-11}\,\mathrm{MeV}^{-2}$
- CKM Matrix Element: $V_{ud} = 0.97419(22)$

  C. Amsler *et al.* (PDG), Phys. Lett. B **667**, 1 (2008)
- Axial Coupling Constant: $g_A = 1.2750(9)$

  H. Abele, Progr. Part. Nucl. Phys., **60**, 1 (2008)

Amplitude of Neutron Decay.
Non-Relativistic Approximation For Baryons

$$M(n \to p + e^- + \tilde{\nu}_e) =$$

$$= -\frac{G_F}{\sqrt{2}} V_{ud} \sqrt{4 m_p m_n} \Big\{ [\bar{u}_e(\vec{k}_e, \sigma_e)\, \gamma^0\, (1 - \gamma^5) v_{\tilde{\nu}_e}(\vec{k}_{\tilde{k}_e}, +\frac{1}{2})]\, [\varphi_p^\dagger\, \varphi_n]$$

$$+ g_A\, [\bar{u}_e(\vec{k}_e, \sigma_e) \vec{\gamma}\, (1 - \gamma^5) v_{\tilde{\nu}_e}(\vec{k}_{\tilde{\nu}_e}, +\frac{1}{2})] \cdot [\varphi_p^\dagger\, \vec{\sigma}\, \varphi_n] \Big\}$$

Continuum-State β^-–Decay Rate of Free Neutron

$$\lambda_{\beta^-_c} =$$

$$= \frac{1}{2m_n}\int (2\pi)^4\delta^{(4)}(k_{\tilde{\nu}_e} + k_e + k_p - k_n)\,\frac{d^3k_p}{(2\pi)^3 2E_p}\,\frac{d^3k_e}{(2\pi)^3 2E_e}\,\frac{d^3k_{\tilde{\nu}_e}}{(2\pi)^3 2E_{\tilde{\nu}_e}}$$

$$\times F(E_e, Z = 1)\,\frac{1}{2}\sum_{\sigma_p,\sigma_e}|M(n \to p + e^- + \tilde{\nu}_e)|^2$$

Continuum-State β^-–Decay Rate of Free Neutron

$$\lambda_{\beta^-_c} = (1+3g^2_A)\,\frac{G^2_F|V_{ud}|^2}{2\pi^3}\,f(Q_{\beta^-_c}, Z=1) = 1.0931(14)\times 10^{-3}\,\mathrm{s}^{-1}$$

Fermi Integral $f(Q_{\beta^-_c}, Z=1)$

$$f(Q_{\beta^-_c}, Z=1) =$$

$$= \int^{Q_{\beta^-_c}+m_e}_{m_e} F(E_e, Z=1)\,(Q_{\beta^-_c}+m_e-E_e)^2\sqrt{E^2_e-m^2_e}\,E_e\,dE_e =$$

$$= \int^{Q_{\beta^-_c}+m_e}_{m_e} \frac{2\pi\alpha E^2_e(Q_{\beta^-_c}+m_e-E_e)^2}{1-e^{-2\pi\alpha E_e/\sqrt{E^2_e-m^2_e}}}\,dE_e = 0.0588\,\mathrm{MeV}^5$$

Lifetime of Neutron Without Radiative Corrections

$$\tau^{(\mathrm{th})}_{\beta^-_c} = 914.8(1.2)\,\mathrm{s} \qquad \tau^{\exp}_{\beta^-_c} = 878.5(8)\,\mathrm{s}$$

Standard $V-A$ Theory of Weak Interactions. Radiative Corrections: A. Sirlin, Phys. Rev. **164**, 1767 (1967); Rev. Mod. Phys. **50**, 573 (1978).

Continuum-State β^-–Decay Rate of Free Neutron

$$\lambda^{(\gamma)}_{\beta^-_c} = (1+3g^2_A)\,\frac{G^2_F|V_{ud}|^2}{2\pi^3}\,f^{(\gamma)}(Q_{\beta^-_c}, Z=1) = 1.1359(14)\times 10^{-3}\,\mathrm{s}^{-1}$$

Fermi Integral $f^{(\gamma)}(Q_{\beta^-_c}, Z=1)$

$$f^{(\gamma)}(Q_{\beta^-_c}, Z=1) = \int^{Q_{\beta^-_c}+m_e}_{m_e} \frac{2\pi\alpha E^2_e(Q_{\beta^-_c}+m_e-E_e)^2}{1-e^{-2\pi\alpha E_e/\sqrt{E^2_e-m^2_e}}}$$

$$\times\left(1+\frac{\alpha}{2\pi}\,g(E_e)\right)dE_e = 0.0611\,\mathrm{MeV}^5$$

Lifetime of Neutron With Radiative Corrections

$$\tau^{(\mathrm{th})}_{\beta^-_c} = 880.1(1.1)\,\mathrm{s} \qquad \tau^{\exp}_{\beta^-_c} = 878.5(8)\,\mathrm{s}$$

Radiative Corrections

$$R_{RC} = \frac{\tilde{f}(Q_{\beta_c^-}, Z = 1)}{f(Q_{\beta_c^-}, Z = 1)} = 1.03912 \qquad R_{RC} = 1.03886(39)$$

Function $g(E_e)$ from Sirlin

$$g(E_e) = g^{(\gamma)}(E_e)_{(1.5\,\%)} + g^{(Z)}(E_e)_{(2.4\,\%)}$$

Proton Recoil Correction

$$\delta\lambda^{(r.c.)}_{\beta^-_c} = (1 + g_A^2)\,\frac{G_F^2|V_{ud}|^2}{2\pi^3}\, f^{(\gamma)}_{r.c.}(Q_{\beta^-_c}, Z = 1)$$

Fermi Integral $f^{(\gamma)}_{r.c.}(Q_{\beta^-_c}, Z = 1)$

$$f^{(\gamma)}_{r.c.}(Q_{\beta^-_c}, Z = 1) = \int^{Q_{\beta^-_c} + m_e}_{m_e} \frac{2\pi\alpha E_e^2 (Q_{\beta^-_c} + m_e - E_e)^2}{1 - e^{-2\pi\alpha E_e/\sqrt{E_e^2 - m_e^2}}}$$

$$\times \Big(1 + \frac{\alpha}{2\pi}\, g(E_e)\Big)\Big\langle \frac{\vec{k}_p^{\,2}}{m_p^2}\Big\rangle\, dE_e = 4.736 \times 10^{-8}\,\mathrm{MeV}^5$$

$$\frac{\delta\lambda^{(r.c)}_{\beta^-_c}}{\lambda^{(\gamma)}_{\beta^-_c}} = \frac{1 + g_A^2}{1 + 3g_A^2}\,\frac{f^{(\gamma)}_{r.c.}(Q_{\beta^-_c}, Z = 1)}{f^{(\gamma)}(Q_{\beta^-_c}, Z = 1)} = 3.463 \times 10^{-7}$$

$V - A$, Scalar and Tensor Weak Interactions

Hamiltonian of Weak Interactions

$$\mathcal{H}_W(x) =$$

$$= \frac{G_F}{\sqrt{2}} V_{ud} \Big\{ [\bar{\psi}_p(x)\gamma_\mu(1 - g_A\gamma^5)\psi_n(x)]\,[\bar{\psi}_e(x)\gamma^\mu(1 - \gamma^5)\psi_{\nu_e}(x)]$$

$$+ g_S\,[\bar{\psi}_p(x)\psi_n(x)]\,[\bar{\psi}_e(x)(1 - \gamma^5)\psi_{\nu_e}(x)]$$

$$+ \frac{1}{2} g_T\,[\bar{\psi}_p(x)\sigma_{\mu\nu}\gamma^5\psi_n(x)]\,[\bar{\psi}_e(x)\sigma^{\mu\nu}(1 - \gamma^5)\psi_{\nu_e}(x)] \Big\}$$

Amplitude of Neutron Decay.
Non-Relativistic Approximation For Baryons

$$M(n \to p + e^- + \tilde{\nu}_e) = -\frac{G_F}{\sqrt{2}} V_{ud} \sqrt{4 m_p m_n}$$

$$\times \Big\{ [\bar{u}_e(\vec{k}_e, \sigma_e)\,(\gamma^0 + g_S)\,(1 - \gamma^5) v_{\tilde{\nu}_e}(\vec{k}_{\tilde{k}_e}, +\frac{1}{2})]\,[\varphi^\dagger_p\,\varphi_n]$$

$$+ [\bar{u}_e(\vec{k}_e, \sigma_e)(g_A + \gamma^0\,g_T)\vec{\gamma}\,(1 - \gamma^5) v_{\tilde{\nu}_e}(\vec{k}_{\tilde{\nu}_e}, +\frac{1}{2})] \cdot [\varphi^\dagger_p\,\vec{\sigma}\,\varphi_n] \Big\}$$

Continuum-State β^-–Decay Rate of Free Neutron

$$\lambda_{\beta^-_c} = \frac{G^2_F|V_{ud}|^2}{2\pi^3}\Big\{(1 + 3g^2_A + g^2_S + 3g^2_T)\, f^{(\gamma)}(Q_{\beta^-_c}, Z = 1) + 2(g_S + 3g_A g_T)\,\tilde{f}^{(\gamma)}(Q_{\beta^-_c}, Z = 1)\Big\}$$

Fermi Integral $\tilde{f}^{(\gamma)}(Q_{\beta^-_c}, Z = 1)$ of Fierz Term

$$\tilde{f}^{(\gamma)}(Q_{\beta^-_c}, Z = 1) = \int^{Q_{\beta^-_c} + m_e}_{m_e} \frac{2\pi\alpha E^2_e(Q_{\beta^-_c} + m_e - E_e)^2}{1 - e^{-2\pi\alpha E_e/\sqrt{E^2_e - m^2_e}}}\Big(\frac{m_e}{E_e}\Big)\, dE_e = \times\Big(1 + \frac{\alpha}{2\pi}\, g(E_e)\Big) = 0.0404\,\mathrm{MeV}^5$$

Electron Energy Spectrum and Angular Distributions

$$\frac{d^5\lambda^{(\gamma)}_{\beta^-_c}}{dE_e d\Omega_e d\Omega_{\tilde{\nu}_e}} = (1 + 3g_A^2 + g_S^2 + 3g_T^2)\,\frac{G_F^2|V_{ud}|^2}{16\pi^5}$$

$$\times\,(Q_{\beta^-_c} + m_e - E_e)^2 E_e \sqrt{E_e^2 - m_e^2}\,F(E_e, Z = 1)\,\Big(1 + \frac{\alpha}{2\pi}\,g(E_e)\Big)$$

$$\times\,\Big(1 + a\,\frac{\vec{k}_e\cdot\vec{k}_{\tilde{\nu}_e}}{E_e E_{\tilde{\nu}_e}} + b\,\frac{m_e}{E_e} + A\,\frac{\vec{\xi}\cdot\vec{k}_e}{E_e} + B\,\frac{\vec{\xi}\cdot\vec{k}_{\tilde{\nu}_e}}{E_{\tilde{\nu}_e}}\Big)$$

Correlation Coefficients

$$a = \frac{1 - g_A^2 - g_S^2 + g_T^2}{1 + 3g_A^2 + g_S^2 + 3g_T^2} \quad \to \quad a = \frac{1 - g_A^2}{1 + 3g_A^2}$$

$$b = 2\,\frac{g_S + 3g_A g_T}{1 + 3g_A^2 + g_S^2 + 3g_T^2} \quad \to \quad b = 2\,\frac{g_S + 3g_A g_T}{1 + 3g_A^2}$$

$$A = -2\frac{g_A(g_A - 1) + g_T(g_S - g_T)}{1 + 3g_A^2 + g_S^2 + 3g_T^2} \quad \to \quad A = -2\frac{g_A(g_A - 1)}{1 + 3g_A^2}$$

$$B = +2\frac{g_A(g_A + 1) + g_T(g_S + g_T)}{1 + 3g_A^2 + g_S^2 + 3g_T^2} + 2\,\frac{g_T + g_A(g_S + 2g_T)}{1 + 3g_A^2 + g_S^2 + 3g_T^2}\,\frac{m_e}{E_e} \to$$

$$\to B = +2\frac{g_A(g_A + 1)}{1 + 3g_A^2} + 2\,\frac{g_T + g_A(g_S + 2g_T)}{1 + 3g_A^2}\,\frac{m_e}{E_e}$$

Correlation Coefficients: Experiment and Theory

Correlation Coefficients	Experiment	Theory
a	$-0.103(4)^{(1)}$	$-0.1065(3)$
b	$-$	$0.0032(23)$
A	$-0.11933(34)^{(1)}$	fit
B	$+0.9821(40)^{(2)}$	$+0.9871(4)_{V-A}$
$C = -0.27484(A+B)$	$-0.2377(26)^{(1)}$	$-0.2385(1)$

- (1) H. Abele, Progr. Part. Nucl. Phys. **60**, 1 (2008)
- (2) A. P. Serebrov *et al.*, J. Exp. Theor. Phys., **113**, 1 (1998)

Lifetime of Neutron: $\tau^{V-A}_{\beta^-_c} = 880.1(1.1)\,\mathrm{s} \to \tau^{\exp}_{\beta^-_c} = 878.5(8)\,\mathrm{s}$

$$b = 2\,\frac{g_S + 3g_A g_T}{1 + 3g_A^2} = 0.0032(23)$$

Neutron Spin–Antineutrino Correlation Coefficient:

$$B_{V-A} = 2\,\frac{g_A(g_A+1)}{1+3g_A^2} = \begin{cases} +0.9821(40) & \textit{Experiment} \\ +0.9871(4) & V-A \textit{ Theory} \end{cases}$$

$$g_T + g_A(g_S + 2g_T) = 0?$$

Scalar g_S and Tensor g_T Coupling Constants

$$g_S = +\frac{b}{2}\,\frac{(1+2g_A)(1+3g_A^2)}{1+2g_A-3g_A^2} = -0.0251(181)$$

$$g_T = -\frac{b}{2}\,\frac{g_A(1+3g_A^2)}{1+2g_A-3g_A^2} = +0.0090(65)$$

$$b = 2\,\frac{g_S + 3g_A g_T}{1 + 3g_A^2} = 0.0032(23) \qquad g_T + g_A(g_S + g_T) = 0$$

Coupling Constants	Experiment	Theory
g_A	1.2750(9)	fit
g_S	$-$	$-0.0251(181)$
g_T	$-$	$+0.0090(65)$

Table: Numerical Values of Weak Coupling Constants

CKM Matrix Element V_{ud}

$$|V_{ud}|^2 = \frac{4910.22}{\tau^{\rm exp}_{\beta^-_c}(1+3g_A^2)} \quad \to \quad |V_{ud}| = \begin{cases} 0.9752(7)\,, & \tau^{\rm exp}_{\beta^-_c} = 878.5(8)\,{\rm s} \\ 0.9713(7)\,, & \tau^{\rm exp}_{\beta^-_c} = 885.7(8)\,{\rm s} \end{cases}$$

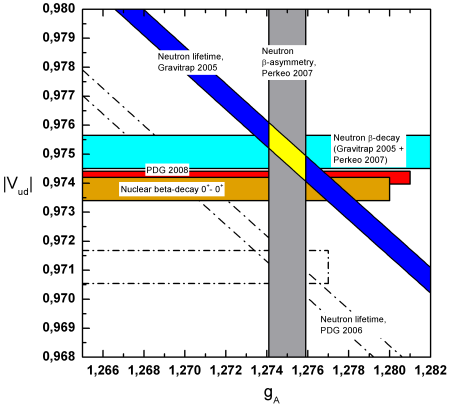

Figure: Dependence of CKM matrix element $|V_{ud}|$ on $\tau_{\beta^-_c}$ and g_A

Summary: We have shown that

- Standard model of electroweak interactions describes well experimental data on
- 1) the continuum-state β^-–decay rate of the neutron, measured by Serebrov *et al.*; $\tau^{\rm exp}_{\beta^-_c} = 878.5(8)\,{\rm s}$ (2008) and
- 2) correlation coefficients of the electron energy spectrum, cited by Abele (2008)
- Contributions of scalar and tensor weak interactions are calculated and can be checked by measuring Fierz term and energy dependent correction to the neutron spin–antineutrino correlation
- They can be also measured from the bound-state β^-–decay of the neutron (see report by Mario Pitschmann)

The results, expounded in this talk, are given in arXiv: 0906.0959 [hep-ph] and obtained in Collaboration with

- Manfried Faber, Atomic Institute of the Austrian Universities, TUWien, Vienna, Austria
- Violetta Ivanova, State Polytechnic University of St. Petersburg, St. Petersburg, Russia
- Johann Marton, Stefan Meyer Institute of Austrian Academy of Sciences, Vienna, Austria
- Mario Pitschmann, Atomic Institute of the Austrian Universities, TUWien, Vienna, Austria
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Thank You For Attention